1-6 Solving Compound and Absolute Value Inequalities

Main Ideas

- Solve compound inequalities.
- Solve absolute value inequalities.

New Vocabulary

compound inequality
intersection
union

GET READY for the Lesson

One test used to determine whether a patient is diabetic is a glucose tolerance test. Patients start the test in a *fasting state*, meaning they have had no food or drink except water for at least 10, but no more than 16, hours. The acceptable number of hours h for fasting can be described by the following compound inequality.

$$h \geq 10 \text{ and } h \leq 16$$

Compound Inequalities A **compound inequality** consists of two inequalities joined by the word *and* or the word *or*. To solve a compound inequality, you must solve each part of the inequality. The graph of a compound inequality containing *and* is the **intersection** of the solution sets of the two inequalities. Compound inequalities involving the word *and* are called *conjunctions*. Compound inequalities involving the word *or* are called *disjunctions*.

Vocabulary Link
Intersection
Everyday Use the place where two streets meet
Math Use the set of elements common to two sets

KEY CONCEPT "And" Compound Inequalities

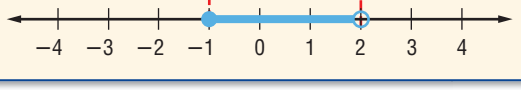
Words A compound inequality containing the word *and* is true if and only if *both* inequalities are true.

Example

$x \geq -1$

$x < 2$

$x \geq -1$ and $x < 2$



Another way of writing $x \geq -1$ and $x < 2$ is $-1 \leq x < 2$.
Both forms are read *x is greater than or equal to −1 and less than 2.*

EXAMPLE 1 Solve an "and" Compound Inequality

Solve $13 < 2x + 7 \leq 17$. Graph the solution set on a number line.

Method 1

Write the compound inequality using the word *and*. Then solve each inequality.

$13 < 2x + 7$ and $2x + 7 \leq 17$
$6 < 2x$ $2x \leq 10$
$3 < x$ $x \leq 5$
$3 < x \leq 5$

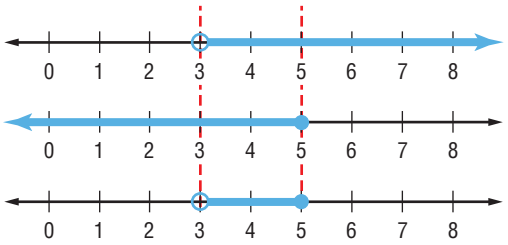
Method 2

Solve both parts at the same time by subtracting 7 from each part. Then divide each part by 2.

$13 < 2x + 7 \leq 17$
$6 < 2x \leq 10$
$3 < x \leq 5$

(continued on the next page)

Graph the solution set for each inequality and find their intersection.



The solution set is $\{x|3 < x \leq 5\}$.

Concepts in Motion
Animation algebra2.com

CHECK Your Progress

1. Solve $8 \leq 3x - 4 < 11$. Graph the solution set on a number line.

The graph of a compound inequality containing *or* is the **union** of the solution sets of the two inequalities.

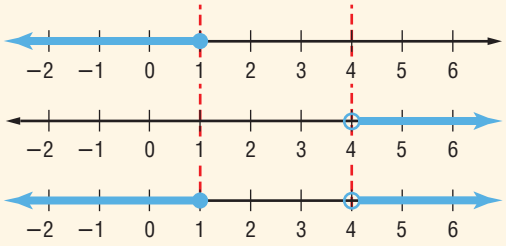
KEY CONCEPT "Or" Compound Inequalities

Words A compound inequality containing the word *or* is true if one or more of the inequalities is true.

Examples $x \leq 1$

$x > 4$

$x \leq 1$ or $x > 4$



Vocabulary Link
Union
Everyday Use something formed by combining parts or members
Math Use the set of elements belonging to one or more of a group of sets

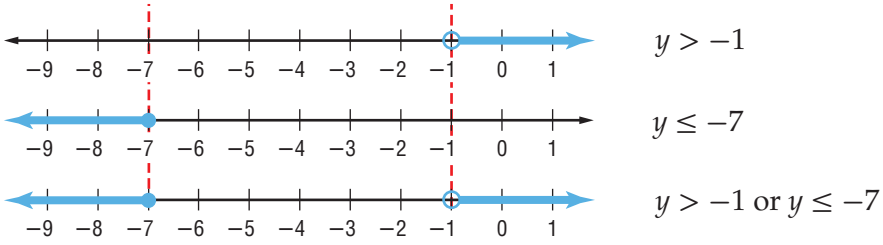
EXAMPLE 2 Solve an "or" Compound Inequality

Solve $y - 2 > -3$ or $y + 4 \leq -3$. Graph the solution set on a number line.

Solve each inequality separately.

$y - 2 > -3$ or $y + 4 \leq -3$

$y > -1$ $\qquad$ $y \leq -7$



The solution set is $\{y|y > -1 \text{ or } y \leq -7\}$.

CHECK Your Progress

2. Solve $y + 5 \leq 7$ or $y - 6 > 2$. Graph the solution set on a number line.

Reading Math

When solving problems involving inequalities,

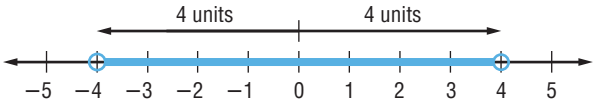
- *within* is meant to be inclusive. Use $\leq$ or $\geq$.
- *between* is meant to be exclusive. Use $<$ or $>$.

Absolute Value Inequalities In Lesson 1-4, you learned that the absolute value of a number is its distance from 0 on the number line. You can use this definition to solve inequalities involving absolute value.

EXAMPLE Solve an Absolute Value Inequality (<)

3 **Solve $|a| < 4$. Graph the solution set on a number line.**

$|a| < 4$ means that the distance between a and 0 on a number line is less than 4 units. To make $|a| < 4$ true, substitute numbers for a that are fewer than 4 units from 0.



Notice that the graph of $|a| < 4$ is the same as the graph of $a > -4$ and $a < 4$.

All of the numbers between -4 and 4 are less than 4 units from 0. The solution set is $\{a \mid -4 < a < 4\}$.

CHECK Your Progress

3. Solve $|x| \leq 3$. Graph the solution set on a number line.

Study Tip

Absolute Value Inequalities

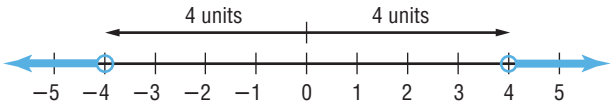
Because the absolute value of a number is never negative,

- the solution of an inequality like $|a| < -4$ is the empty set.
- the solution of an inequality like $|a| > -4$ is the set of all real numbers.

EXAMPLE Solve an Absolute Value Inequality (>)

4 **Solve $|a| > 4$. Graph the solution set on a number line.**

$|a| > 4$ means that the distance between a and 0 on a number line is greater than 4 units.



Notice that the graph of $|a| > 4$ is the same as the graph of $\{a > 4$ *or* $a < -4\}$.

The solution set is $\{a \mid a > 4 \text{ or } a < -4\}$.

CHECK Your Progress

4. Solve $|x| \geq 3$. Graph the solution set on a number line.

An absolute value inequality can be solved by rewriting it as a compound inequality.

KEY CONCEPT *Absolute Value Inequalities*

Symbols For all real numbers a and b, $b > 0$, the following statements are true.
1. If $|a| < b$, then $-b < a < b$.
2. If $|a| > b$, then $a > b$ or $a < -b$

Examples If $|2x + 1| < 5$, then $-5 < 2x + 1 < 5$
If $|2x + 1| > 5$, then $2x + 1 > 5$ or $2x + 1 < -5$.

These statements are also true for $\leq$ and $\geq$, respectively.

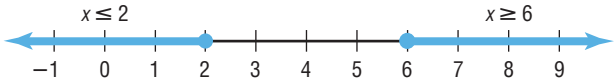
EXAMPLE Solve a Multi-Step Absolute Value Inequality

5 **Solve $|3x - 12| \geq 6$. Graph the solution set on a number line.**

$|3x - 12| \geq 6$ is equivalent to $3x - 12 \geq 6$ or $3x - 12 \leq -6$.
Solve the inequality.

$3x - 12 \geq 6$	or	$3x - 12 \leq -6$	Rewrite the inequality.
$3x \geq 18$		$3x \leq 6$	Add 12.
$x \geq 6$		$x \leq 2$	Divide by 3.

The solution set is $\{x \mid x \geq 6 \text{ or } x \leq 2\}$.



CHECK Your Progress

5. Solve $|3x + 4| < 10$. Graph the solution set on a number line.

Real-World Link

When executives in a recent survey were asked to name one quality that impressed them the most about a candidate during a job interview, 32 percent said honesty and integrity.

Source: careerexplorer.net

Real-World EXAMPLE Write an Absolute Value Inequality

6 **JOB HUNTING** **To prepare for a job interview, Megan researches the position's requirements and pay. She discovers that the average starting salary for the position is $38,500, but her actual starting salary could differ from the average by as much as $2450.**

a. Write an absolute value inequality to describe this situation.
Let x equal Megan's starting salary.

Her starting salary could differ from the average	by as much as	$2450.
$\lvert 38{,}500 - x \rvert$	$\leq$	2450

b. Solve the inequality to find the range of Megan's starting salary.
Rewrite the absolute value inequality as a compound inequality. Then solve for x.

$$\begin{aligned} -2450 \leq\ & 38{,}500 - x & \leq 2450 \\ -2450 - 38{,}500 \leq\ & 38{,}500 - x - 38{,}500 & \leq 2450 - 38{,}500 \\ -40{,}950 \leq\ & -x & \leq -36{,}050 \\ 40{,}950 \geq\ & x & \geq 36{,}050 \end{aligned}$$

The solution set is $\{x \mid 36{,}050 \leq x \leq 40{,}950\}$. Thus, Megan's starting salary will fall within $36,050 and $40,950.

CHECK Your Progress

6. The ideal pH value for water in a swimming pool is 7.5. However, the pH may differ from the ideal by as much as 0.3 before the water will cause discomfort to swimmers or damage to the pool. Write an absolute value inequality to describe this situation. Then solve the inequality to find the range of acceptable pH values for the water.

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CHECK Your Understanding

Examples 1–5 (pp. 41–44)

Solve each inequality. Graph the solution set on a number line.

1. $3 < d + 5 < 8$

2. $-4 \leq 3x - 1 < 14$

3. $y - 3 > 1$ or $y + 2 < 1$

4. $p + 6 < 8$ or $p - 3 > 1$

5. $|a| \geq 5$

6. $|w| \geq -2$

7. $|h| < 3$

8. $|b| < -2$

9. $|4k - 8| < 20$

10. $|g + 4| \leq 9$

Example 6 (p. 44)

11. FLOORING Deion is considering several types of flooring for his kitchen. He estimates that he will need between 55 and 60 12-inch by 12-inch tiles to retile the floor. The table below shows the price per tile for each type of tile Deion is considering.

Tile Type	Price per Tile
Vinyl	\$0.99
Slate	\$2.34
Porcelain	\$3.88
Marble	\$5.98

Write a compound inequality to determine how much he could be spending.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12, 13	1
14, 15	2
16, 17	3
18, 19	4
20, 21	5
22, 23	6

Solve each inequality. Graph the solution set on a number line.

12. $9 < 3t + 6 < 15$

13. $-11 < -4x + 5 < 13$

14. $3p + 1 \leq 7$ or $2p - 9 \geq 7$

15. $2c - 1 < -5$ or $3c + 2 \geq 5$

16. $|g| \leq 9$

17. $|3k| < 0$

18. $|2m| \geq 8$

19. $|b - 4| > 6$

20. $|3w + 2| \leq 5$

21. $|6r - 3| < 21$

SPEED LIMITS **For Exercises 22 and 23, use the following information.**
On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.

22. Write an inequality to represent the allowable speed for a car on an interstate highway.

23. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway.

Solve each inequality. Graph the solution set on a number line.

24. $-4 < 4f + 24 < 4$

25. $a + 2 > -2$ or $a - 8 < 1$

26. $|-5y| < 35$

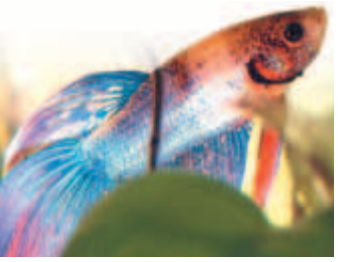
27. $|7x| + 4 < 0$

28. $|n| \geq n$

29. $|n| \leq n$

30. $\frac{|2n - 7|}{3} \leq 0$

31. $\frac{|n - 3|}{2} < n$



Real-World Link

Adult Male Size: 3 inches

Water pH: 6.8–7.4

Temperature: 75–86°F

Diet: omnivore, prefers live foods

Tank Level: top dweller

Difficulty of Care: easy to intermediate

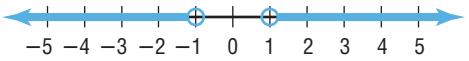
Life Span: 2–3 years

Source: www.about.com

32. FISH A Siamese Fighting Fish, better known as a Betta fish, is one of the most recognized and colorful fish kept as a pet. Using the information at the left, write a compound inequality to describe the acceptable range of water pH levels for a male Betta.

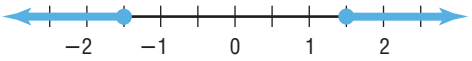
Write an absolute value inequality for each graph.

33.



34.

35.



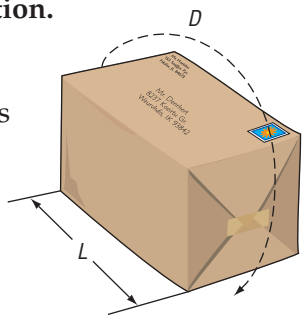
36.

37.

38.

39. HEALTH *Hypothermia* and *hyperthermia* are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both conditions are potentially dangerous and occur when a person's body temperature fluctuates by more than 8° from the normal body temperature of 98.6°F. Write and solve an absolute value inequality to describe body temperatures that are considered potentially dangerous.

MAIL For Exercises 40 and 41, use the following information.
The U.S. Postal Service defines an oversized package as one for which the length L of its longest side plus the distance D around its thickest part is more than 108 inches and less than or equal to 130 inches.



40. Write a compound inequality to describe this situation.

41. If the distance around the thickest part of a package you want to mail is 24 inches, describe the range of lengths that would classify your package as oversized.

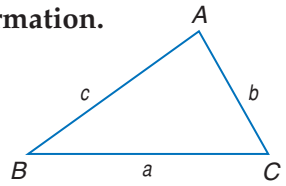
AUTO RACING For Exercises 42 and 43, use the following information.
The shape of a car used in NASCAR races is determined by NASCAR rules. The rules stipulate that a car must conform to a set of 32 templates, each shaped to fit a different contour of the car. The biggest template fits over the center of the car from front to back. When a template is placed on a car, the gap between it and the car cannot exceed the specified tolerance. Each template is marked on its edge with a colored line that indicates the tolerance for the template.

42. Suppose a certain template is 24.42 inches long. Use the information in the table at the right to write an absolute value inequality for templates with each line color.

43. Find the acceptable lengths for that part of a car if the template has each line color.

Line Color	Tolerance (in.)
Red	0.07
Blue	0.25
Green	0.5

GEOMETRY **For Exercises 44 and 45, use the following information.**
The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.



44. Write three inequalities to express the relationships among the sides of $\triangle ABC$.

45. Write a compound inequality to describe the range of possible measures for side c in terms of a and b. Assume that $a > b > c$. (*Hint:* Solve each inequality you wrote in Exercise 44 for c.)

Graphing Calculator

LOGIC MENU **For Exercises 46–49, use the following information.**
You can use the operators in the LOGIC menu on the TI-83/84 Plus to graph compound and absolute value inequalities. To display the LOGIC menu, press 2nd [TEST] ▶.

46. Clear the Y= list. Enter $(5x + 2 > 12)$ and $(3x - 8 < 1)$ as Y1. With your calculator in DOT mode and using the standard viewing window, press GRAPH. Make a sketch of the graph displayed.

47. Using the TRACE function, investigate the graph. Based on your investigation, what inequality is graphed?

48. Write the expression you would enter for Y1 to find the solution set of the compound inequality $5x + 2 \geq 3$ or $5x + 2 \leq -3$. Then use the graphing calculator to find the solution set.

49. A graphing calculator can also be used to solve absolute value inequalities. Write the expression you would enter for Y1 to find the solution set of the inequality $|2x - 6| > 10$. Then use the graphing calculator to find the solution set. (*Hint:* The absolute value operator is item 1 on the MATH NUM menu.)

EXTRA PRACTICE
See pages 892, 926.
Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

50. **OPEN ENDED** Write a compound inequality for which the graph is the empty set.

51. **FIND THE ERROR** Sabrina and Isaac are solving $|3x + 7| > 2$. Who is correct? Explain your reasoning.

Sabrina

$|3y + 7| > 2$

$3y + 7 > 2$ or $3y + 7 < -2$

$3y > -5$ $\quad$ $3y < -9$

$y > -\frac{5}{3}$ $\quad$ $y < -3$

Isaac

$|3x + 7| > 2$

$-2 < 3x + 7 < 2$

$-9 < 3x < -5$

$-3 < x < -\frac{5}{3}$

52. **CHALLENGE** **Graph each set on a number line.**

a. $-2 < x < 4$

b. $x < -1$ or $x > 3$

c. $(-2 < x < 4)$ and $(x < -1$ or $x > 3)$ (*Hint:* This is the intersection of the graphs in part **a** and part **b**.)

d. Solve $3 < |x + 2| \leq 8$. Explain your reasoning and graph the solution set.

53. ***Writing in Math*** Use the information about fasting on page 41 to explain how compound inequalities are used in medicine. Include an explanation of an acceptable number of hours for this fasting state and a graph to support your answer.

STANDARDIZED TEST PRACTICE

54. ACT/SAT If $5 < a < 7 < b < 14$, then which of the following best describes $\frac{a}{b}$?

A $\frac{5}{7} < \frac{a}{b} < \frac{1}{2}$

B $\frac{5}{14} < \frac{a}{b} < \frac{1}{2}$

C $\frac{5}{7} < \frac{a}{b} < 1$

D $\frac{5}{14} < \frac{a}{b} < 1$

55. REVIEW What is the solution set of the inequality $-20 < 4x - 8 < 12$?

F $-7 < x < 1$

G $-3 < x < 5$

H $-7 < x < 5$

J $-3 < x < 1$

Spiral Review

Solve each inequality. Then graph the solution set on a number line. (Lesson 1-5)

56. $2d + 15 \geq 3$ **57.** $7x + 11 > 9x + 3$ **58.** $3n + 4\,(n + 3) < 5(n + 2)$

59. CONTESTS To get a chance to win a car, you must guess the number of keys in a jar to within 5 of the actual number. Those who are within this range are given a key to try in the ignition of the car. Suppose there are 587 keys in the jar. Write and solve an equation to determine the highest and lowest guesses that will give contestants a chance to win the car. (Lesson 1-4)

Solve each equation. Check your solutions. (Lesson 1-4)

60. $5\,|x - 3| = 65$ **61.** $|2x + 7| = 15$ **62.** $|8c + 7| = -4$

Name the property illustrated by each statement. (Lesson 1-3)

63. If $3x = 10$, then $3x + 7 = 10 + 7$.

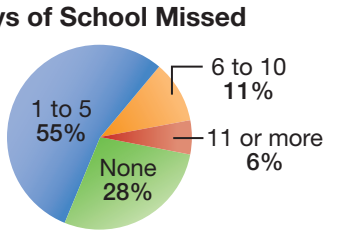
64. If $-5 = 4y - 8$, then $4y - 8 = -5$.

65. If $-2x - 5 = 9$ and $9 = 6x + 1$, then $-2x - 5 = 6x + 1$.

SCHOOL For Exercises 66 and 67, use the graph at the right.

66. Illustrate the Distributive Property by writing two expressions to represent the number of students at a high school who missed 5 or fewer days of school if the school enrollment is 743.

67. Evaluate the expressions from Exercise 66.



Source: Centers for Disease Control and Prevention

Simplify each expression. (Lesson 1-2)

68. $6a - 2b - 3a + 9b$ **69.** $-2(m - 4n) - 3(5n + 6)$

Find the value of each expression. (Lesson 1-1)

70. $6(5 - 8) \div 9 + 4$ **71.** $(3 + 7)^2 - 16 \div 2$ **72.** $\frac{7(1 - 4)}{8 - 5}$